

PERIODIC TEST - II (2025 - 26) QUESTION PAPER

Subject: Mathematics (041)_Band 3

Marks: 25

Name: _____

Grade: XII

Time: 1 Hour

Date: 21-07-2025

General Instructions:

- This question paper contains five sections A, B, C, D and E. Each section is compulsory.
- There is no overall choice in the paper.
- Section A comprises 4 Multiple Choice Questions of 1 mark each.
- Section B comprises 3 Very Short Answer Type Questions of 2 marks each.
- Section C comprises 2 Short Answer Type Questions of 3 marks each.
- Section D comprises 1 Long Answer Type Question of 5 marks.
- Section E has 1 Case Study, in which 2 Very Short Answer Type Questions are of 1 mark each and 1 Short Answer Type Question is of 2 marks. An internal choice is provided in the 2-mark question of the Case Study.

Section A

Question Nos. 1 to 4 carry 1 mark each.

1. A spherical balloon has a variable diameter $\frac{3}{2}(2x+1)$. The rate of change of its volume with respect to x is _____.
 (a) $\frac{27\pi}{8}(2x+1)^2$ (b) $\frac{9}{4}\pi(2x+1)^3$ (c) $\frac{9\pi}{16}(2x+1)^3$ (d) $\pi(2x+1)^2$
2. If the function $f(x) = x^2 e^{-x}$, then ' f ' increases in the interval _____.
 (a) $(-\infty, \infty)$ (b) $(-2, 0)$ (c) $(2, \infty)$ (d) $(0, 2)$
3. Let $A = [0, 1]$ and N be the set of natural numbers. Then the mapping $f: N \rightarrow A$, defined by $f(2n-1) = 0$, $f(2n) = 1$ for all $n \in N$, is _____.
 (a) one-one (b) onto (c) bijection (d) one-one and into
4. The domain of $f(x) = \cos^{-1}(x^2 - 4)$ is _____.
 (a) $[3, 5]$ (b) $[0, \pi]$ (c) $[-\sqrt{5}, -\sqrt{3}] \cap [-\sqrt{5}, \sqrt{3}]$ (d) $[-\sqrt{5}, -\sqrt{3}] \cup [\sqrt{3}, \sqrt{5}]$

Section B

Question Nos. 5 to 7 carry 2 marks each.

5. At what point of the ellipse $16x^2 + 9y^2 = 400$, does the ordinate decrease at the same rate at which the abscissa increases?
6. Draw the graph of $\tan^{-1} x$.
7. Write the following function in its simplest form:

$$y = \sin^{-1} \left(\frac{\sqrt{1+x} + \sqrt{1-x}}{2} \right)$$

Section C

Question Nos. 8 and 9 carry 3 marks each.

8. If $y = x^x + (\sin x)^x$, then find $\frac{dy}{dx}$.

9. If $x = a \cos \theta + b \sin \theta$, $y = a \sin \theta - b \cos \theta$, then show that $x^2 \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + y = 0$.

Section D

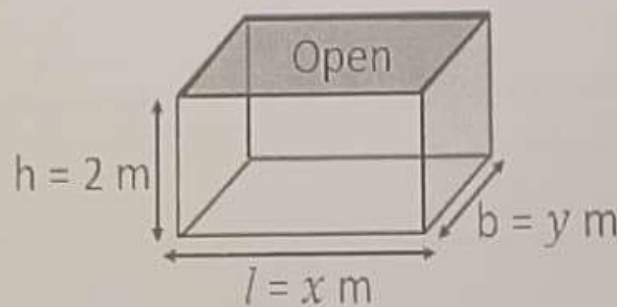
Question No. 10 carries 5 marks.

10. Let $A = \{x \in \mathbb{Z} : 0 \leq x \leq 12\}$. Show that the relation $R = \{(a, b) : a, b \in A, |a - b| \text{ is divisible by } 4\}$ is an equivalence relation. Find the set of all elements related to 1. Also, write the equivalence class [2].

Section E

Question No. 11 is a Case - Based Question that has three sub parts. Sub - parts (i) and (ii) are compulsory and carry one mark each. Sub-part (iii) carries 2 marks and has an internal choice.

11. A tank with a rectangular base and rectangular sides and open at the top, is to be constructed using a metal sheet in such a way that its depth is 2 m and volume is 8 m^3 . Building the tank costs ₹ 70 per square metre for the base and ₹ 45 per square metre for the sides.



Based on the above information, answer the following questions:

- Find the total cost of building the tank, expressed as a function of x .
- Find the relation between the length and width of the least expensive tank.
- What is the cost of the least expensive tank?

OR

- Find the size of the metal sheet required to construct the least expensive tank.