Half Yearly Examination (2025 – 2026)
Subject: Mathematics
Date:18/08/2025
Time: 3 hours
Max Marks: 80

Name: ____________
Class: 12 Division: ______
Roll No: ____________
Invigilator's Sign ____________

The Orbis School

General Instructions:-

Read the following instructions very carefully and strictly follow them:

(i) This Question paper contains 38 questions. All questions are compulsory.
(ii) Question paper is divided into five sections - Section A, B, C, D and E.
(iii) In Section **A- Question no. 1 to 18 are multiple choice questions (MCQs)** and Question no.19 and 20 are assertion –Reason based questions of 1 mark each.
(iv) In Section **B- Question no.21 to 25** are Very Short Answer (VSA)-type questions, carrying **2 marks** each.
(v) In Section **C - Question no.26 to 31** are Short Answer (SA)-type questions, carrying **3 marks** each.
(vi) In Section **D - Question no.32 to 35** are Long Answer (LA)-type questions, carrying **5 marks** each.
(vii)In Section **E - Question no.36 to 38** are case based questions, carrying **4 marks** each
(viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in section C, 3 questions in Section D and 1 subpart each in 2 questions of Section E.
(ix) Use of calculator is not allowed.

SECTION A

(This Section comprises of multiple choice questions (MCQs) of 1 mark each)

Select the correct option (Question 1 – Question 18):

Q1. If P, Q, and PQ are matrices of order 3 x 2, a x b and 3 x 4 respectively then the number of elements of the matrix Q is?

(a) 6 (b) 8 (c) 4 (d) 12

Q2. The function, $f(x) = x|x|$, at x=0 is:

(a) Continuous and differentiable (b) Continuous but not differentiable
(c) Differentiable but not Continuous (d) Neither differentiable nor Continuous

Q3. If $f(x) = \sqrt{7g(x) - 3}$, g (3) = 4 and g' (3) = 5, find f' (3).

(a) $\frac{1}{2}$ (b) $\frac{5}{2}$ (c) $\frac{7}{2}$ (d) $\frac{3}{2}$

Q4. On which of the following intervals is the function f given by $f(x) = x^{100} + \sin x - 1$ decreasing?

(a) (0, 1) (b) $(\frac{\pi}{2}, \pi)$ (c) $(0, \frac{\pi}{2})$ (d) None of these

Q5. If $f(x)=\begin{cases} \frac{kx}{|x|} & \text{if } x < 0 \\ 3 & \text{if } x \geq 0 \end{cases}$, is continuous at x =0, then the value of k is

(a) -3 (b) 0 (c) 3 (d) any real number

Q6. If the inverse of the matrix $\begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ is the matrix $\begin{bmatrix} 1 & 3 & 3 \\ 1 & p & 3 \\ 1 & 3 & 4 \end{bmatrix}$ then the value of p is

(a) 3 (b) 4 (c)1 (d) 0

Q7. The value of |A|, if A= $\begin{bmatrix} 0 & 2x-1 & \sqrt{x} \\ 1-2x & 0 & 2\sqrt{x} \\ -\sqrt{x} & \sqrt{x} & 0 \end{bmatrix}$ where $x \in R^{+}$, is

(a) $(2x+1)^2$ (b) 0 (c) $(2x+1)^3$ (d) $(2x-1)^2$

Q8. If area of triangle is 35 sq. units with vertices (2, 6), (5, 4) and (k, 4). Then k is

(a) 12 (b)-2 (c) -12,-2 (d) 12, -2

Q9. The set of all points where the function f(x) = x+|x| is differentiable, is

(a) $(0, \infty)$ (b) $(-\infty, 0)$ (c) $(-\infty, 0) \cup (0, \infty)$ (d) $(-\infty,\infty)$

Q10. If $xe^{y} = 1$, then the value of $\frac{dy}{dx}$ at x=1 is

(a)-1 (b) 1 (c) –e (d) e

Q11. *The linear inequalities or equations or restrictions on the variables of a linear programming problem are called:*

(a) *a constraint* (b) Decision variables (c) Objective function (d) None of the above

Q12. The domain of the function y=$\cos^{-1}\frac{1}{x-3}$

(a) [-1, 1] (b)[2, 4] (c) $(-\infty, 2] \cup [4, \infty)$ (d) $(-\infty, 1] \cup [1, \infty)$

Q13. Let A={3,5} . Then, number of reflexive relations on A is

(a) 2 (b) 4 (c) 0 (d) 8

Q14. The value of $\cos\left[2\tan^{-1}\frac{1}{2}\right]$

(a) $\frac{3}{5}$ (b) $\frac{4}{5}$ (c) $\frac{5}{4}$ (d) 1

Q15. In equation $y - x \leq 0$ represents

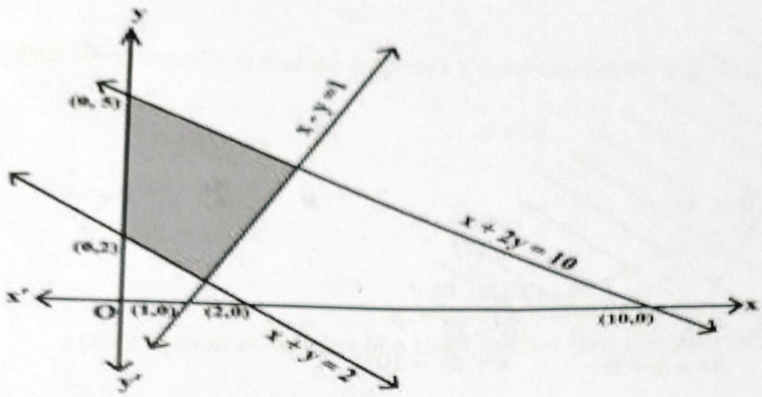
(a) The half plane that contains the positive x-axis
(b) Closed half plane above the line y = x, which contains positive y-axis
(c) Half plane that contains the negative x-axis
(d) None of these

$0 < \frac{1}{x-3} < 1$

$2 < x-3$

$4 < x$

Q16. The feasible region corresponding to the linear constraints of a Linear Programming Problem is given below.



Which of the following is not a constraint to the given Linear Programming Problem?

(a) $x+y \geq 2$ (b) $x+2y \leq 10$ (c) $x-y \geq 1$ (d) $x-y \leq 1$

Q17. Given that A is a square matrix of order 3 and $|A|=-2$, then $|adj(2A)|$ is equal to

(a) -2^6 (b) $+4$ (c) -2^8 (d) 2^8

Q18. The amount of pollution content added in air in a city due to x-diesel vehicles is given by $P(x) = 0.005x^3+0.02x^2+ 30x$. The marginal increase in pollution content when 3 diesel vehicles are added

(a) 30 units (b) 30.255 units (c) 31 units (d) 31.255 units

$0.015x^2 + 0.04x + 30$

135 $0.135 + 0.12 + 30$

ASSERTION – REASON BASED QUESTIONS

For questions 19 and 20, two statements are given – one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below:

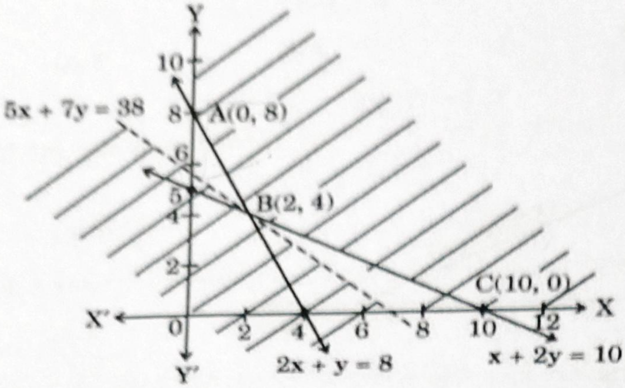
(a) Both A and R are true and R is the correct explanation of the assertion

(b) Both A and R are true but R is not the correct explanation of the assertion

(c) A is true, but R is false

(d) A is false, but R is true

Q19. Assertion (A): The shaded portion of the graph represents the feasible region for the given Linear Programming Problem (LPP).



Min Z = 50x + 70y

subject to constraints

$2x + y \geq 8$, $x + 2y \geq 10$, $x, y \geq 0$

Z= 50x + 70y has a minimum value = 380 at B(2, 4).

Reason (R): The region representing $50x + 70y < 380$ does not have any point common with the feasible region.

Q20. ASSERTION (A): The relation f:{1,2,3,4} → {x, y, z, p} defined by f ={(1,x),(2,y),(3,z)} is a bijective function.

REASON (R): The function f:{1,2,3}→{x,y,z,p} such that f ={(1,x),(2,y),(3,z)} is one-one.

SECTION B

(This Section comprises of 5 very short answer (VSA) type questions of 2 marks each)

Q21. The radius of a cylinder is decreasing at a rate of 2 cm/s and the altitude is increasing at the rate of 3 cm/s. Find the rate of change of volume of this cylinder when its radius is 4 cm and altitude is 6 cm.

Q22. Given A = $\begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$, compute A^{-1} and show that $2A^{-1} = 9I - A$. $2\begin{pmatrix} 7 & 4 \\ 3 & 2 \end{pmatrix}$

OR

If A = $\begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ be such that $A^{-1} = kA$, then find the value of k. $\begin{pmatrix} 14 & 8 \\ 6 & 4 \end{pmatrix}$

Q23.Let X={2,6,12,20} and Y={2,4,7,10}

(i) Define a bijective function from set X to set Y

(ii) Show that the function you defined is one-one .

Q24. If a= $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right) + \cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ and b=$\tan^{-1}(\sqrt{3}) - \cot^{-1}\left(-\frac{1}{\sqrt{3}}\right)$ then find the value of a+b.

Q25. Differentiate $\frac{\sin x}{\sqrt{\cos x}}$ with respect to x

OR

Find the values of k so that the function f is continuous at the indicated point

f(x)= $\begin{cases} kx+1 & if\ x \le \pi \\ \cos x & if\ x > \pi \end{cases}$, at x= π

SECTION C

(This Section comprises of 6 short answer (SA) type questions of 3 marks each)

Q26. If x = a $\sec^3\theta$, y = a $\tan^3\theta$, then find $\frac{d^2y}{dx^2}$ at $\theta=\frac{\pi}{4}$.

OR

Show that the function f(x) = |x-3|, x ∈ R, is continuous but not differentiable at x = 3.

Q27. If A= $\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ find $(AB)^{-1}$

Q28. Find the intervals in which the function f given by f(x) = sin x + cos x, 0 ≤ x ≤ 2π is increasing or decreasing.

Q29. Solve the following Linear Programming Problem graphically:

Minimize: z = x+2y,

subject to the constraints: x + 2y ≥ 100, 2x - y ≤ 0,2x + y ≤200 x, y ≥ 0.

OR

Solve the following Linear Programming Problem graphically:

Maximise :Z=-x+2y, subject to the constraints: x ≥ 3, x + y ≥ 5, x + 2y ≥ 6, y ≥ 0.

Q30. Show that y = log(1+x) - $\frac{2x}{2+x}$, x > -1 is an increasing function of x, throughout its domain.

OR

A wire of length 28 m is to be cut into two pieces. One of the pieces is to be made into a square and the other into a circle. What should be the length of the two pieces so that the combined area of the square and the circle is minimum?

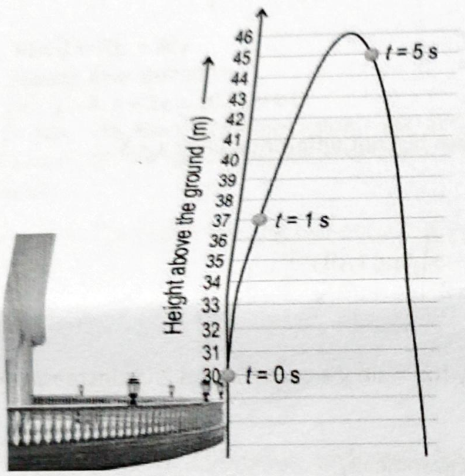
Q31. If $(a+bx)e^{\frac{y}{x}} = x$ then prove that $x\frac{d^2y}{dx^2} = \left(\frac{a}{a+bx}\right)^2$

SECTION D

(This Section comprises of 4 long answer (LA) type questions of 5 marks each)

Q32. A ball is thrown from a balcony. Its height above the ground after t seconds is given by $h(t)=pt^2+qt+r$, where p, q, and r $\in R$ and h(t) is in meters.

Shown below is the trajectory of the ball with its height from the ground at t=0 s, t=1 s and t= 5 s.



Solve for p, q, r and h(t) using the matrix method. Show your work and give valid reasons.

Q33. A function f: $[0, \infty) \rightarrow [-5,\infty)$ be defined by $f(x) = 4x^2+4x-5$. Prove that the function is a one-one and onto function.

OR

Show that the function f: $(-\infty, 0) \rightarrow (-1, 0)$ defined by $f(x) = \frac{x}{1+|x|}$, $x \in (-\infty, 0)$ bijective function.

Q34. Find the intervals on which the function $f(x)=(x-1)^3(x-2)^2$ is:

(i) Strictly increasing (ii) Strictly decreasing

Q35. (a) For a positive constant 'a', differentiate $a^{t+\frac{1}{t}}$ with respect to $\left(t+\frac{1}{t}\right)^{a}$ where t is a non-zero real number.

OR

(b) Find $\frac{dy}{dx}$ if $y^x + x^y + x^x = a^b$, where a and b are constants.

SECTION E

(This section comprises of 3 case-study/passage-based questions of 4 marks each with sub-parts. The First 2 case study questions have three sub-parts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two sub-parts of 2 marks each.)

Q36. CASE STUDY – I

Ramesh, the owner of a sweet selling shop, purchased some rectangular card board sheets of dimension 25 cm by 40 cm to make container packets without top. Let x cm be the length of the side of the square to be cut out from each corner to give that sheet the shape of the container by folding up the flaps.

Based on the above information answer the following questions.

(i) Express the volume (V) of each container as function of x only. [1 Mark]

(ii) Find $\frac{dv}{dx}$ [1 Mark]

(iii) (a) For what value of x, the volume of each container is maximum? [2 Marks]

OR

(b) Check whether V has a point of inflection at $x = \frac{65}{6}$ or not? [2 Marks]

Q37. CASE STUDY – II

A carpenter needs to make a wooden cuboidal box, closed from all sides, which has a square base and fixed volume. Since he is short of the paint required to paint the box on completion, he wants the surface area to be minimum.

On the basis of the above information, answer the following questions:

(i) Taking length = breadth = x m and height = y m, express the surface area (S) of the box in terms of x and its volume (V), which is constant. [1 Mark]

(ii) Find $\frac{ds}{dx}$ [1 Mark]

(iii) (a) Find a relation between x and y such that the surface area (S) is minimum. [2 Marks]

OR

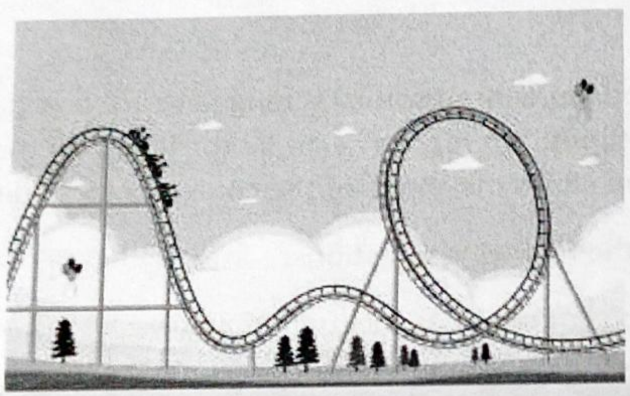
(b) If surface area (S) is constant, the volume $(V) = \frac{1}{4}(Sx - 2x^3)$, x being the edge of base. Show that volume (V) is maximum for $x = \sqrt{\frac{S}{6}}$ [2 Marks]

Q38. CASE STUDY – III

Read the following passage and answer the questions given below.

The equation of the path traced by a roller-coaster is given by the polynomial $f(x) = a(x+9)(x+1)(x-3)$.

If the roller-coaster crosses y-axis at a point (0, -1), answer the following questions.



(i) Find the value of 'a'. [2 Marks]

(ii) Find $f''(x)$ at $x = 1$. [2 Marks]

$-77a = 0$

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